

# Quadratic inequalities

## A LEVEL LINKS

Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions)

## Key points

- First replace the inequality sign by = and solve the quadratic equation.
- Sketch the graph of the quadratic function.
- Use the graph to find the values which satisfy the quadratic inequality.

## Examples

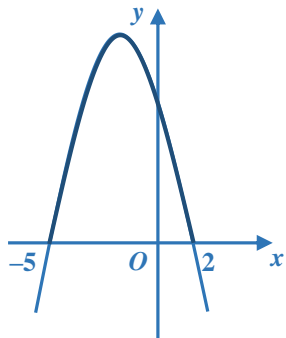
**Example 1** Find the set of values of  $x$  which satisfy  $x^2 + 5x + 6 > 0$

<p> <math>x^2 + 5x + 6 = 0</math>  <math>(x + 3)(x + 2) = 0</math>  <math>x = -3</math> or <math>x = -2</math> </p> <p> </p> <p> <math>x &lt; -3</math> or <math>x &gt; -2</math> </p>	<ol style="list-style-type: none"> <li>1 Solve the quadratic equation by factorising.</li> <li>2 Sketch the graph of <math>y = (x + 3)(x + 2)</math></li> <li>3 Identify on the graph where <math>x^2 + 5x + 6 &gt; 0</math>, i.e. where <math>y &gt; 0</math></li> <li>4 Write down the values which satisfy the inequality <math>x^2 + 5x + 6 &gt; 0</math></li> </ol>
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**Example 2** Find the set of values of  $x$  which satisfy  $x^2 - 5x \leq 0$

<p> <math>x^2 - 5x = 0</math>  <math>x(x - 5) = 0</math>  <math>x = 0</math> or <math>x = 5</math> </p> <p> </p> <p> <math>0 \leq x \leq 5</math> </p>	<ol style="list-style-type: none"> <li>1 Solve the quadratic equation by factorising.</li> <li>2 Sketch the graph of <math>y = x(x - 5)</math></li> <li>3 Identify on the graph where <math>x^2 - 5x \leq 0</math>, i.e. where <math>y \leq 0</math></li> <li>4 Write down the values which satisfy the inequality <math>x^2 - 5x \leq 0</math></li> </ol>
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**Example 3** Find the set of values of  $x$  which satisfy  $-x^2 - 3x + 10 \geq 0$

$-x^2 - 3x + 10 = 0$ $(-x + 2)(x + 5) = 0$ $x = 2 \text{ or } x = -5$  $-5 \leq x \leq 2$	<ol style="list-style-type: none"> <li><b>1</b> Solve the quadratic equation by factorising.</li> <li><b>2</b> Sketch the graph of <math>y = (-x + 2)(x + 5) = 0</math></li> <li><b>3</b> Identify on the graph where <math>-x^2 - 3x + 10 \geq 0</math>, i.e. where <math>y \geq 0</math></li> <li><b>3</b> Write down the values which satisfy the inequality <math>-x^2 - 3x + 10 \geq 0</math></li> </ol>
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## Practice

- 1** Find the set of values of  $x$  for which  $(x + 7)(x - 4) \leq 0$
- 2** Find the set of values of  $x$  for which  $x^2 - 4x - 12 \geq 0$
- 3** Find the set of values of  $x$  for which  $2x^2 - 7x + 3 < 0$
- 4** Find the set of values of  $x$  for which  $4x^2 + 4x - 3 > 0$
- 5** Find the set of values of  $x$  for which  $12 + x - x^2 \geq 0$

## Extend

Find the set of values which satisfy the following inequalities.

- 6**  $x^2 + x \leq 6$
- 7**  $x(2x - 9) < -10$
- 8**  $6x^2 \geq 15 + x$

## Answers

1  $-7 \leq x \leq 4$

2  $x \leq -2$  or  $x \geq 6$

3  $\frac{1}{2} < x < 3$

4  $x < -\frac{3}{2}$  or  $x > \frac{1}{2}$

5  $-3 \leq x \leq 4$

6  $-3 \leq x \leq 2$

7  $2 < x < 2\frac{1}{2}$

8  $x \leq -\frac{3}{2}$  or  $x \geq \frac{5}{3}$